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AMPLITUDES OF LINEAR OSCILLATIONS DETERMINED BY LINEAR HOMOGENOUS DIFFERENTIAL EQUATION OF THE SECOND ORDER AND LIOUVILLE-BESGE FORMULAE

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Abstract. In this paper we present some quadraturic aspects of solving the equation by means of quadratures. It has a special comparative value when estimating Sturm's zeros, "periods" and variable amplitudes in cases which are solvable by means other than quadratures (iterations).

1. Introduction and Preliminaries

It is well known how hard it is to solve the equation of ordinary (nonharmonic) oscillations of the second order $y''(x) + a(x)y(x) = 0$, $a(x) > 0$ by means of quadratures. Long time ago, Besge and Liouville anticipated that some classes of the equation could be solved by means of quadratures, but it was not until L. M. Berkovich [3], who did it. Problems on zeros of the solution (Sturm's theorems), the problem of distance between successive zeros (being a replacement for nonexistent periods), as well as the problems of amplitudes of solutions, are still not solved in a satisfactory way (for details see [4], [5], [9], [10], [12], [13] and [14]).

It is shown in our paper [7] for the equation of linear oscillations

$$y''(x) + a(x)\,y(x) = 0, \;\; a(x) > 0 \tag{1.1}$$

Key words and phrases. Linear differential equation of the second order, Quadraturally- oscillatory solutions, Variable amplitude, Relation between amplitude and frequency.

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